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Interpretation of failure under cyclic contact loading

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Failure of ceramic components under cyclic loading—often designated as cyclic fatigue—is of high importance in lifetime prediction. Many ceramics show this effect and mostly the crack growth rate $\mathrm{d}a/\mathrm{d}N$ (a = crack depth, N = number of cycles) can be described by a power law relation

$$\frac{\mathrm{d}a}{\mathrm{d}N} = A\left(\frac{\Delta K}{K_{\mathrm{Ic}}}\right)^{\mathrm{n}} \tag{1}$$

with ΔK = variation of the applied stress intensity factor, K_{Ic} = fracture toughness, and the crack growth parameters A and n. If the exponent n is small, the investigated material is very sensitive to cyclic crack growth. For high n-values ($n \to \infty$), no significant fatigue crack growth effect occurs. Therefore, it is of high importance to know this exponent for a given material.

Under the assumption of the inert strength σ_{c} and the lifetime under cyclic loading being caused by the same flaw population, integration of Equation 1 leads to the number of cycles to failure N_{f}

$$N_{\mathrm{f}} = \frac{B\sigma_{\mathrm{c}}^{\mathrm{n}-2}}{(\Delta\sigma)^{\mathrm{n}}} \tag{2}$$

with a specific material parameter B. In a plot of log $\Delta\sigma$ or $\log(\sigma_{\max})$ versus $\log(N_{\mathrm{f}})$, a so-called Wöhler plot, a straight line with a slope of $-1/n$ is expected.

A second method to determine n is the evaluation of the Weibull distribution of cycles to failure N_{f} and the Weibull distribution of inert strength σ_{c}. If the Weibull distribution of strength is given by

$$F(\sigma_{\mathrm{c}}) = 1 - \exp\left[-\left(\frac{\sigma_{\mathrm{c}}}{\sigma_0}\right)^{\mathrm{m}}\right] \tag{3}$$

the scatter in lifetime can be written as

$$F(N_{\mathrm{f}}) = 1 - \exp\left[-\left(\frac{N_{\mathrm{f}}}{N_0}\right)^{\mathrm{m}^*}\right] \tag{4}$$

with

$$m^* = \frac{m}{n-2} \tag{5}$$

The parameter n can be obtained from the Weibull parameters m and m^* for strength and lifetime by applying (5).

Strength and fatigue measurements were performed on Al_2O_3 ceramics [1, 2]. In one test series, load was applied in opposite cylinder loading tests (Fig. 1) [3]. Two cylinders of 8 mm in diameter, made of hardened steel, were pressed onto the rectangular specimen with a periodically varying force P resulting in a contact stress $\sigma^* = 0.49P/(tW)$. Weibull parameters were determined for strength and lifetime and the constants of a power law for cyclic crack growth were evaluated from the Weibull parameters as well as from the Wöhler diagram [1].

In Fig. 2 the fatigue results from [1] (frequency $f = 10$ Hz and an R-ratio of $R = \sigma_{\min}/\sigma_{\max} = 0.05$) are given as Wöhler diagrams. The median values are indicated by the crosses. From these plots, we find for the median values of the cycles to failure

$$\hat{N}_{\mathrm{f}} = \frac{C}{\sigma_{\max}^{\mathrm{n}}} \tag{6}$$

$C = 8.2 \times 10^{27}\,\mathrm{MPa}^{9.5}$ for F99.7 and $C = 1.8 \times 10^{32}\,\mathrm{MPa}^{11}$ for F99.9, and the exponents n as compiled in Table I. The exponents n of the Weibull evaluations were determined by Equation 5 and are also given in Table I.

The results found in the contact fatigue tests are in strong contrast to results from cyclic 4-point bending tests [2]. For the identical batch of material F99.7, the latter revealed $n \approx 25$, that is by a factor of 3–4 higher than in the contact fatigue tests. The intention of this paper is to interpret these strong differences.

A friction effect is considered for coarse-grained alumina in tension-tension cyclic loading [4]. The fatigue effect is influenced by periodical friction between the grain-interlock bridges existing in such materials. "Wear" decreases the effect of the intergranular frictional contact points, resulting in a loss of traction at the junction, reducing the crack tip shielding, and increasing the effective load at the crack tip. This effect superimposed by normal environmentally assisted crack growth (as occurring in quasi-static tests) then allows for an interpretation of the fatigue behavior under tension-tension loading.

From dynamic contact strength tests performed under different loading rates, it was found that no environmentally assisted crack growth effect occurs [5]. Fig. 3a shows results obtained for alumina F99.7 in dynamic 4-point bending tests. They clearly exhibit a subcritical crack growth effect. The result of "dynamic contact loading tests" is given in Fig. 3b. In [5] this effect was explained to be a consequence of the negative K_{I} contribution at the location of fracture. Due to $K_{\mathrm{I}} < 0$, the crack is closed near the tip at least and water diffusion is inhibited in this region. The effect of a closed

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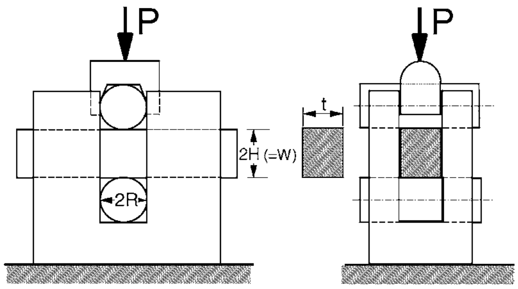

Figure 1 A two-roller test device for contact strength tests and Hertzian pressure distribution.

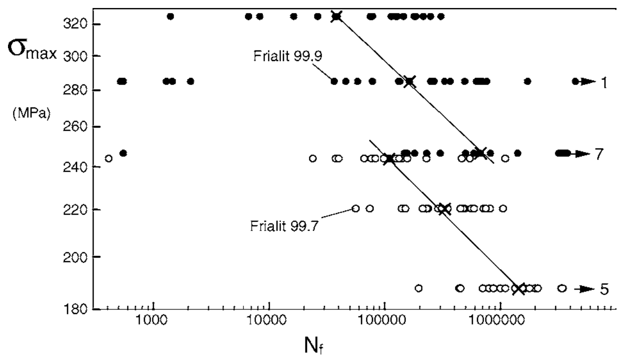

Figure 2 Wöhler diagram of the contact fatigue results ($f = 10$ Hz, $R \cong 0.05$).

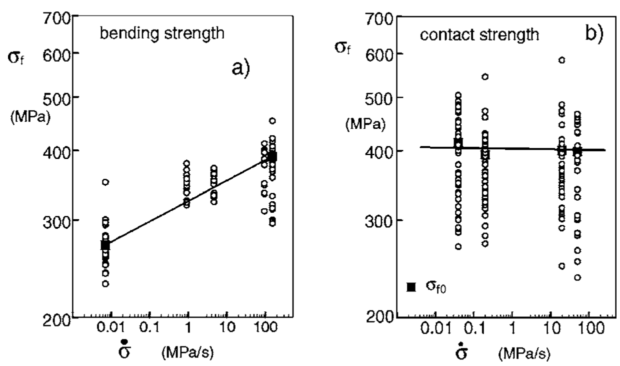

Figure 3 Dynamic bending strength: (a) 4-point bending tests, and (b) contact strength tests, squares: characteristic strength values σ_0.

crack is also present under cyclic loading conditions. Consequently, the effect of quasi-static subcritical crack growth can be neglected.

For the interpretation of the fatigue behavior under contact load, a frictional effect also is assumed to be responsible, namely, periodical crack face sliding.

Near the end of the Hertzian contact zone, where the natural cracks start to propagate, the mode-I stress intensity factor K_I is negative (see Fig. 4a). The condition $K < 0$ ensures that the crack either is totally closed or at least partially closed with the crack faces being in contact in the crack tip region (Fig. 4b). The effective stress intensity factor represents a pure mode-II loading $K_{II,eff}$ which is equal to the applied stress intensity factor K_{II} reduced by a friction term μK_I, which in case of the Richard [6] mixed-mode fracture criterion yields [7]

$$K_{eff} = \sqrt{\frac{3}{2}} K_{II,eff}, \quad K_{II,eff} = K_{II} + \mu K_I \tag{7}$$

with the friction coefficient μ.

Under cyclic mode-II loading, the crack faces rub on each other. This may lead to a smoothing of surface roughness and a reduction of the friction coefficient with increasing number of cycles, i.e.,

$$\mu = f(\Delta K_{II}, K_I, N) \tag{8}$$

As a simple set-up, we use an exponential decrease

$$\mu = \mu_0 \exp(-N/N_0) \tag{9}$$

where μ_0 is the initial friction coefficient and N_0 a characteristic number of cycles which depends on $\Delta\sigma$ and possibly on R. Introducing Equation 9 into Equation 7 yields for the effective stress intensity factor

$$K_{eff} = \sqrt{\frac{3}{2}} [K_{II} + \mu_0 K_I \exp(-N/N_0)] \tag{10}$$

where μ_0 is the initial friction coefficient at $N = 0$. Equation 10 shows that the effective stress intensity factor increases with an increasing number of cycles. Spontaneous crack extension and, consequently, failure of a specimen occurs, if the effective stress intensity factor equals fracture toughness K_{Ic}, i.e., for $K_{eff} = K_{Ic}$ with the number of cycles to failure N_f given by

$$K_{Ic} = \sqrt{\frac{3}{2}} [K_{II} + \mu_0 K_I \exp(-N_f/N_0)] \tag{11}$$

and

$$K_{II} \geq \sqrt{\frac{2}{3}} K_{Ic} \tag{12}$$

Contact strength also results from (7) by setting $K_{eff} = K_{Ic}$ or from (11) by setting $N_f = 0$

$$K_{Ic} = \sqrt{\frac{3}{2}} \left[K_{II}^{(c)} + \mu_0 K_I^{(c)}\right] \tag{13}$$

where the applied stress intensity factors in the critical case are denoted by the superscript (c). Combining (10) and (13) yields for the number of cycles to failure

$$N_f = N_0 \ln \frac{\mu_0 K_I}{K_{II}^{(c)} - K_{II} + \mu_0 K_I^{(c)}} \tag{14}$$

TABLE I Exponents n from cyclic contact loading tests [1], last column: 4-point bending tests [2]

Material	n, Equation 5	n, Equation 6	n, [2]
Alumina F99.7	9.5	6.55	25
Alumina F99.9	11.0	19.9	

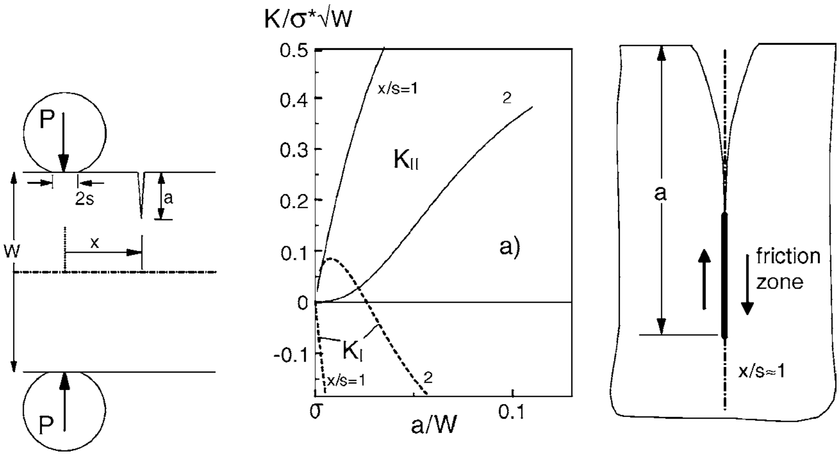

Figure 4 (a) Mode-I and mode-II stress intensity factors near the Hertzian contact area and (b) crack-face contact zone.

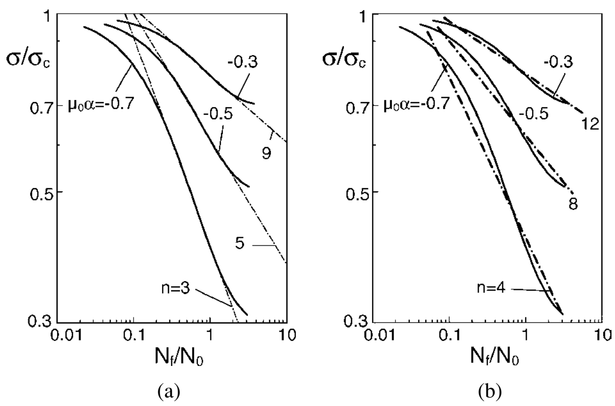

Figure 5 Wöhler-like representation of Equation 16, (a) straight-line fit of the linear parts and (b) straight-line fit of larger curve segments.

If for a fixed crack length a, the ratio of the two stress intensity factor contributions is expressed as

$$\frac{K_{\mathrm{I}}}{K_{\mathrm{II}}} = \frac{K_{\mathrm{I}}^{(\mathrm{c})}}{K_{\mathrm{II}}^{(\mathrm{c})}} = \alpha(a), \quad \alpha < 0, \quad \frac{K_{\mathrm{I}}}{K_{\mathrm{I}}^{(\mathrm{c})}} = \frac{K_{\mathrm{II}}}{K_{\mathrm{II}}^{(\mathrm{c})}} = \frac{\sigma}{\sigma_{\mathrm{c}}} \tag{15}$$

where σ is the applied contact stress and σ_{c} the contact strength, it holds

$$N_{\mathrm{f}} = N_0 \ln \frac{\mu_0 \alpha \frac{\sigma}{\sigma_{\mathrm{c}}}}{1 + \mu_0 \alpha - \frac{\sigma}{\sigma_{\mathrm{c}}}} \tag{16}$$

This relation is plotted in Fig. 5 together with the fitting lines determined from the linear parts (Fig. 5a) and from the extended fit range $N_{\mathrm{f}}/N_0 \approx 0.04$–4 (Fig. 5b). The fitted n-values are in the range of the experimentally observed ones.

From the considerations made above, it can be concluded that delayed failure in contact fatigue tests is caused by a reduction of the friction term due to cyclic loading and by a related increase of the effective stress intensity factor until fracture toughness is reached.

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